

THE HEATING COEFFICIENTS OF RHEOSTATS AND THE
CALCULATION OF RESISTANCES FOR CURRENTS
OF SHORT AND MODERATE DURATIONS.

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STEFAN'S LAW OF RADIATION.—Stefan's law of radiation for black or grey bodies in vacuum reads—

$$P_R = AS (\delta_f^4 - \delta_b^4) \quad \dots \quad 1.$$

where S_c is the radiating surface, δ_f and δ_b , the absolute temperatures of heated body and surroundings respectively, and A a coefficient which varies with the nature of the surface and the surrounding media.

When δ_f differs little from δ_b , *i. e.* when the temperature rise is small, we may write

$$P_R = 4AS \delta_b^3 (\delta_f - \delta_b) = A'S_c \delta \quad \dots \quad 2.$$

where $\delta = \delta_f - \delta_b$ = temperature size.

When δ_f is very great, δ_b may be neglected, and

$$P_R = AS_c \delta_f^4 \quad \dots \quad 3.$$

For electrical apparatus these formulæ hold very approximately only, equation 2 being the one which is generally used for machines and rheostats. The coefficient A' , which may be called the emissivity, or the heat radiated per unit cooling surface for a temperature difference of one degree, is thereby assumed constant and independent of the temperature.

Test results indicate that A' is by no means a constant quantity if formula 2 is used. Where the air is stationary, formula 1 is more correct, *i. e.* the power radiated varies still with a high power of the temperature. For pure radiation the ratios of the powers radiated according to equations 1 and 2 are given in fig. 1, for temperatures up to 500° C., which clearly indicate that the use of formula 2 may give results which are altogether unreliable, as soon as the temperature rise exceeds a few degrees, whenever the power radiated varies with a high power of the temperature.

When a current of I amperes flows through a resistance of R ohms the heat generated in unit time is

preferable to employ the approximate formula 2 *assuming* A' *to be constant for the purpose of integration only.* We then obtain

$$\frac{d\delta}{dt} = \frac{i^2\rho sl - A'Pl\delta}{CGsl} 7.$$

Integrated

[illegible]

This is the well-known law of the heating curve.

The integration holds for constant A' and ρ . The variation of ρ for most resistance materials is negligible, but that of A' may be 20 or more per cent. To be able to integrate we should use an average value or proceed in steps. In practice it is the maximum temperature rise, which is of importance, for which A' may be more easily assumed correctly.

When $P_H = P_R$ no further rise takes place, and we get

$$P_H = P_R = A'S_C(\delta - \delta_c) = A'S_C\delta_m \quad (9)$$

whence

$$\delta_m = \frac{P_H}{A'S_G} = \frac{I^2_{\rho l}}{A'Pl_s} = \frac{I^2_p}{A'Ps} = \frac{i^2_{ps}}{A'P} \quad . \quad . \quad . \quad . \quad . \quad . \quad 10.$$

and

$$\delta = \delta_m \left\{ 1 - \epsilon^{-\frac{A'Pt}{CGs}} \right\} \quad . \quad . \quad . \quad . \quad 11.$$

Let

$$t_o = \frac{CG_s}{A'P} = \frac{CG_{sl}}{A'Pl} = \frac{CG_{sl}\delta}{A'Pl\delta} = \frac{\text{heat required to reach final temp.}}{\text{rate of generation of heat}} \quad . \quad 12.$$

=heat required to reach actual final temperature if there were no cooling.

Owing to the latter, the actual temperature rise only reaches $\left(1 - \frac{1}{\epsilon}\right)$, or 63·4 per cent. of its steady value in time t_0 . We call t_0 the time constant. Hence

$$\delta = \delta_m \left\{ 1 - \varepsilon^{-\frac{t}{t_0}} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad 13.$$

It will be obvious from these equations that the accuracy of the pre-calculation of the temperature rise of a rheostat, or the size of its conductors for a given rise and specified load conditions, will entirely depend upon the correct assumption of A' or $\frac{1}{A'}$. These can, however, be obtained from tests alone, and even then great care has to be exercised, as A' may vary considerably when simply turning the rheostat over, whereby the ventilation is affected.

In figs. 2, 3, 4, and 5 values of A' and $\frac{1}{A'}$ have been plotted from tests carried out.

For fig. 2 the resistances consisted of spiral wires of No. 22 S.W.G. thickness (0.71 mm.), wound on mandrils $\frac{3}{8}$ in. thick, and freely suspended

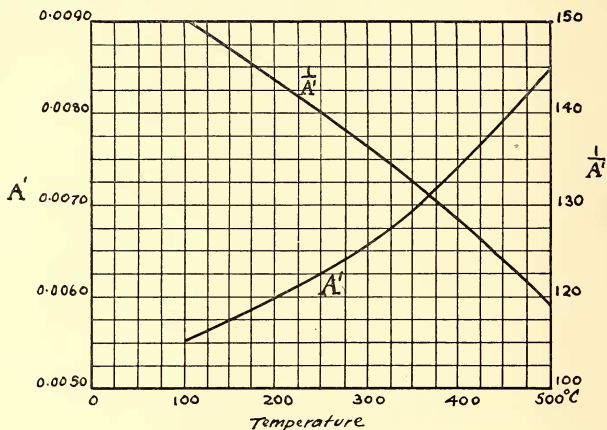


Fig. 2.

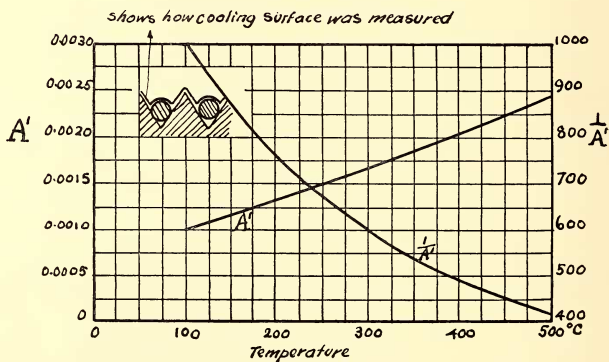


Fig. 3.

between the ends, giving excellent ventilation. We see that with an increase in the temperature the emission of heat improves, $\frac{1}{A'}$ dropping from 150 at

100° to 117 at 500°. The suspension of coils in this manner gives the best cooling arrangement; but the mechanical construction is weak, as the wires sag.

In fig. 3 the heating coefficient is given for the same wire, which is now wound on porcelain rods 1 in. in diameter and provided with a standard

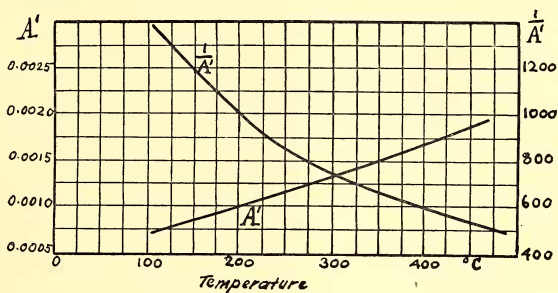


Fig. 4

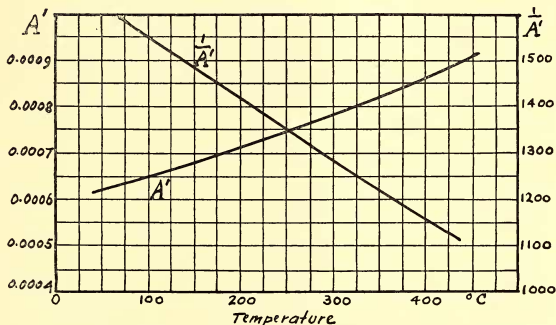


Fig. 5.

screw-thread twelve to the inch. The total cooling surface has been measured for the evaluation of A' in the manner indicated in the inset. We see that now a much greater cooling surface is required to emit the same amount of heat, $\frac{1}{A'}$ being 1000, whereas for freely suspended coils it was 150 for the same temperature—viz. 100° C.

Fig. 4 gives the coefficients for asbestos-woven rheostats, forming sheets, about $\frac{3}{8}$ in. apart, the area of the sheets being taken for the evaluation of A' .

Fig. 5 holds for a grid resistance, the cross section of the conductor being 9×5 mms.² The grids were closely packed, the distance between the conductors being about equal to their thickness. As was to be expected, the emission of heat was rather poor, and for high temperatures very irregular for different parts of the grid. The curves give average values.

In all experiments the rheostats were quite open at the top, so that the hot air could get away. When the asbestos-woven resistance and the grids were turned over, so that the top was shielded by an iron plate, the temperature rose much higher, showing that a generalisation of values for heating coefficients is absurd. For every particular type data should be obtained experimentally, and the position and manner of fixing should be clearly stated.

The heating coefficient A' , being a function of the temperature, may be expressed by an equation. For fig. 2 we get

$$A' = 0.0026\delta^{0.158} \quad \text{whence—}$$

$$\delta_m = \sqrt[1.158]{\frac{P_H}{0.0026S_C}} \quad \text{Compared with Stefan's law we get}$$

$$= \sqrt[1.158]{\frac{385P_H}{S_C}} \quad P_R = AS_C(\delta_f^{0.04} - \delta_s^{0.04}) \text{ where } A \text{ is now constant}$$

and δ_f and δ_s are absolute temperatures.

On the whole, it will be more convenient to use equation 10, and to take A' , or better $\frac{1}{A'}$, from the curves.

$\frac{1}{A'}$, indicates the size of the cooling surface per watt lost for 1° C. rise.

RHEOSTATS FOR LOADS OF SHORT DURATION.—The time in which a resistance reaches its final temperature for a given load depends upon the heat capacity and the ventilation. The freely suspended coils used for fig. 2 required $2\frac{1}{2}$ minutes, when wound on the porcelain rods 45 minutes, while the cast-iron grid rheostat took 92 minutes. This time does not vary much with the load, but the time constant does, as A' increases with the temperature. It will thus be impossible to say offhand what constitutes a load of short duration. For freely suspended coils of thin wire even a minute is a load of long duration, as its temperature will then be near the maximum, whereas for the same wire wound on porcelain the cooling effect during this time is negligible. Oil-insulated rheostats do not reach their steady temperature until hours have passed.

The initial rise is expressed by

$$\begin{aligned} \frac{d\delta}{dt} &= \frac{\text{rate of generation of heat}}{\text{thermal capacity}} \\ &= \frac{I^2 \rho s l}{C G s l} = K I^2 \end{aligned} \quad 14.$$

if we assume ρ and C constant, which is approximately correct for constantan and manganin, but not for iron, copper, etc. The temperature coefficient for iron grids is, however, usually much lower than for ordinary iron.

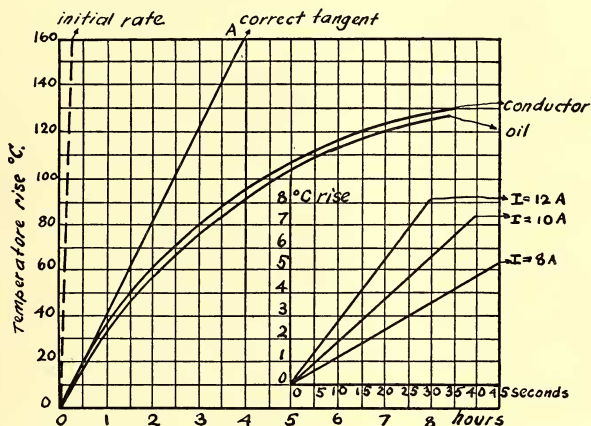


Fig. 6.

Equation 14 holds for bare conductors. For those wound on insulating material or embedded in oil, sand, or enamel we have

$$\begin{aligned} \frac{d\delta}{dt} &= \frac{I^2 \rho s l}{\frac{1}{2} (C G s l) \times s^2} \\ &= \frac{I^2 R}{\frac{1}{2} (M C)} \end{aligned} \quad 15.$$

where M is the weight and C the specific heat of the materials used.

Of importance is the case when conductors are immersed in oil, for which we have

$$\frac{d\delta}{dt} = \frac{I^2 R}{M C + M_o C_o} \quad 15a.$$

When designing a starter with oil insulation for short duration starting, the question arises, How much oil is to be taken into account? Evidently

only a small portion is heated during the time required to start a motor, as there is no time for circulation to set in. Experiments carried out with freely suspended coils of No. 22 S.W.G. copper wire showed that the time elapsing before circulation takes place depends upon the current density. The results of the test are shown in fig. 6. When the current was 8 amperes it took 70 seconds, for 10 amperes 40, and for 12 amperes 30 seconds. The rates of temperature rise during these times are also indicated in the inset on the right. The quantity of oil affected was approximately the same in all cases—1.25 kilograms out of 24 kilograms contained in the vessel. A large vessel for short duration starters thus means waste of material.

Analysing the results, it was found that for the same temperature rise in air-cooled and oil-insulated rheostats for short duration loads, when cooling is negligible, the current density in the latter case may be about four times higher than in the former. It must, however, be remembered that for air-cooled resistances much higher temperature rises are allowable, depending upon the mechanical design, whereas the maximum temperature of oil should not exceed 100° C. if sludging and deterioration is to be avoided. The saving of material in oil-insulated starters is thus illusory, even if we neglect the cost of the vessel and the expense of the oil. As a matter of fact, the oil is used mainly for safety in places where sparking is dangerous, as in coal-mines, where all switching has to take place under oil.

For rheostats embedded in sand or enamel the current density may be about twice as high as for air-cooled resistances, assuming the same temperature rise, which may be considerably higher than oil permits, so that now resistance material is saved.

RHEOSTATS FOR LOADS OF MODERATE DURATION.—The general law of heating reads

$$\delta = \delta_m \left(1 - \epsilon - \frac{t}{t_o} \right)$$

As all heating curves have the same shape we may plot one for $\delta_m = 1$ and $t_o = 1$, as has been done in fig. 7. We next determine t_o and the ratio $\frac{t}{t_o}$, whence for a given value of δ_m the rise is also determined.

$$\left. \begin{aligned} \delta_m &= \frac{I^2 \rho}{A' P_s} \text{ generally} \\ &= \frac{4I^2 \rho}{A' d^3} \text{ for circular conductors} \end{aligned} \right\} \quad . \quad . \quad . \quad 16.$$

$$\left. \begin{aligned} t_o &= \frac{CGs}{A' P} \text{ generally} \\ &= \frac{CGd}{4A'} \text{ for circular conductors} \end{aligned} \right\} \quad . \quad . \quad . \quad 17.$$

where d denotes the diameter.

We see that the accuracy of the calculation depends again upon the correct assumption of A' or $\frac{1}{A'}$. But this quantity depends itself upon the temperature, and as the latter is unknown, A' is unknown. For a new design there is nothing left but to assume A' and to make several trial calculations until A' and δ correspond.

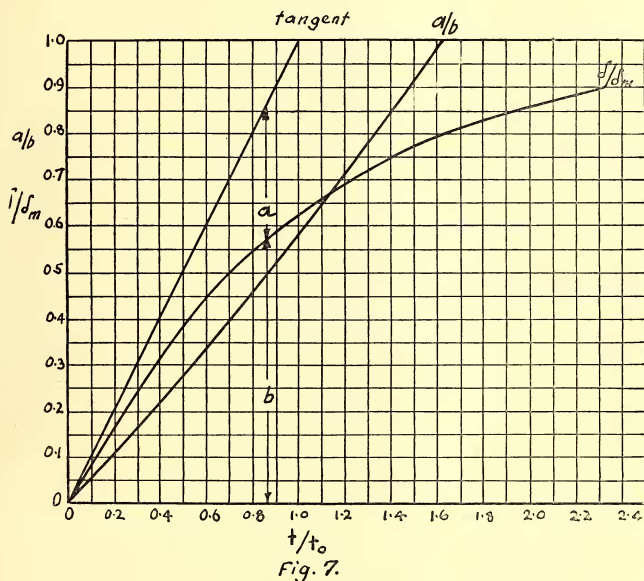


Fig. 7.

When a resistance has been constructed, it is best to find t_0 experimentally, as the calculated value is always too small, except perhaps for freely suspended wires with little heat capacity and excellent ventilation. A heat run is thus essential. For small rheostats this does not take long; but for large apparatus the time required and the cost of a heat run may be considerable. The time may, however, be reduced as follows:

We carry out a test and plot the temperature rise against time until the curve commences to bend towards the abscissa axis. This time should not exceed one or one and a half hours even for the largest rheostats. In fig. 7 a second curve has been plotted, which gives the ratio

$$\frac{\text{temp. rise if there were no cooling} - \text{actual temp. rise}}{\text{actual temp. rise}} = \frac{a}{b}$$

as function of $\frac{t}{t_o}$. The temperature rise which neglects cooling is given by the tangent to the curve, hence with $\frac{a}{b}$ known, we take from the figure the corresponding ratio $\frac{t}{t_o}$, and then from the first curve the value $\frac{\delta}{\delta_m}$, whence δ_m is determined.

The accuracy of this method depends upon the correct drawing of the tangent. The latter can in most cases not be drawn from the initial temperature rises, unless the heat capacity is very small. This is clearly shown in fig. 6, in which the initial rate is given by the dotted line on the left, whereas the correct tangent lies far to the right, for reasons explained previously. The tangent must therefore be guessed as correctly as possible, which is not very difficult if one draws a complete set of tangents starting where the experiment was interrupted and moving towards the origin, thus enveloping the heating curve with tangents. Once the tangent in the origin is known, the construction of the whole heating curve is simple, and clearly indicated in fig. 7, which is self-explanatory.

The experimental values of t_o usually lie between 0 and about 50, according to the size and construction of the resistance. For the grid rheostat the calculated value was 12 minutes, the experimental one 17 and 19 minutes for maximum temperatures of 290° and 210° C. respectively.

For a known maximum rise the time constant follows from—

$$t_o = \frac{t}{\ln \left(\frac{\delta_m}{\delta_m - \delta} \right)}$$

so that only the rise δ in the time t has to be noticed.